AN INTERFACE FOR THE FDTD DIAKOPTICS

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Abstract **--- A new formulation of the TLM type (or the directional wave type) interface for the FDTD diakoptics is proposed. The interface is implemented in the FDTD algorithm using the concept of the impedance boundary condition incorporated with a signal source model without adopting the absorbing boundary condition. Thus efficient and rigorous analyses are accomplished.**

I. INTRODUCTION

Diakoptics is considered to be a key to the efficient analysis of large-scale structures. In this approach, a structure is divided into parts that can be analyzed individually. Thus the method is suitable for parallel computations using a multi-CPU machine or independent computers connected by networks.

The diakoptics can be formulated both in the time and frequency domains. The time domain diakoptics is advantageous for the analysis of circuits that include nonlinear parts and has been discussed in connection with the time domain methods of numerical field simulations [1]-[5]. Some original works have been done using the TLM method [1]-[3]. In the TLM algorithm, the discrete Green function can be evaluated naturally in a form of directional voltage wave impulse responses. On the other hand, the implementation in the FDTD method is not as simple as in the TLM method. This is because the total field is dealt with in the FDTD instead of the directional fields. Emulation of the TLM type interface in the FDTD was studied by Huang et. al. [5], who proposed a recursive algorithm for separating the total field into directional waves. However, they required the use of a perfect absorbing boundary condition (ABC) for absorbing the impulse waves, which degraded the efficiency and utility of the method. They also proposed a total-field type interface and applied it to one-dimensional problems, but extending this type to two- and three-dimensional problems is not straightforward in fact.

This paper presents a new interface for the FDTD diakoptics. The interface is completely equivalent to the TLM type. Nevertheless, the ABC is not necessary. Instead, we use the impedance boundary condition (IBC). This enables efficient and rigorous calculations. The interface presented here may be considered the time domain version of that described in [6].

II. FORMULATION

Let us start with a one-dimensional example for simplicity. Figure 1 represents a transmission line model of plane waves traveling in the x direction. The infinitely long line is loaded with a signal source at the center and is divided into three segments as shown. Two interfaces are defined: one is between segments (i) and (ii); the other between (i) and (iii). In our formulation, the removed nodes are terminated by a standard impedance after dividing the segments. Directional waves which are used to define the Green function are derived using the terminating conditions. In the following, we will obtain a discrete Green function that characterizes segments (ii) and (iii), and then calculate the response of the whole structure based on the diakoptics approach.

II-I. Calculation of the discrete Green function

Figure 2 illustrates segment (ii). A voltage source with an internal impedance of R is newly added to terminate the removed node at the interface. Suppose that the field has E_y and H_z components only, which correspond to nodal voltage and the current of the transmission line, respectively. The directional waves defined in the figure at the interface are expressed in terms of the nodal voltage and current as

$$v_{in}(t) = \frac{V(t) - R\ I(t)}{2}, \quad (1)$$

$$v_{out}(t) = \frac{V(t) + R\ I(t)}{2}. \quad (2)$$

The terminating condition is represented by

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$$V(t) - R\ I(t) = V_s(t). \quad (3)$$

From (1)-(3), we get

$$v_{in}(t) = \frac{V_s(t)}{2}, \quad (4)$$

$$v_{out}(t) = V(t) - \frac{V_s(t)}{2}. \quad (5)$$

Equations (4) and (5) are the basis of subsequent derivation. We define the Green function $g(t)$ as the response $v_{out}(t)$ of the impulse excitation $v_{in}(t) = \delta(t{=}0)$, which completely characterizes segment (ii). $g(t)$ can be calculated by the following procedure.

#1 Excite segment (ii) using voltage source $v_s(t)$ such as $v_s(t) = 2\delta(t{=}0)$.

#2 Find the response of nodal voltage $V(t)$ by time domain simulation.

#3 Calculate $g(t)$ by $g(t) \equiv v_{out}(t) = V(t) - \delta(t{=}0)$.

The terminating condition can be implemented in the FDTD algorithm using the concept of the IBC incorporated with a signal source model, which was proposed in [6]. According to the phantom-known type formulation given in [6], the FDTD formula of the IBC for $E_y(0)$ in Fig. 2 is

$$E_y^n(0) = \frac{\varepsilon_0 \varepsilon_{ry} - \Delta t/R\Delta x}{\varepsilon_0 \varepsilon_{ry} + \Delta t/R\Delta x} E_y^{n-1}(0) - \frac{2\Delta t}{(\varepsilon_0 \varepsilon_{ry} + \Delta t/R\Delta x)\Delta x} H_z^{n-1/2}(1/2) - \frac{2\Delta t}{(\varepsilon_0 \varepsilon_{ry} + \Delta t/R\Delta x)\Delta x} \left(\frac{E_s^n + E_s^{n-1}}{2R} \right). \quad (6)$$

In this discrete system, the value at time $t = n\Delta t$ is denoted by superscript n. The last term in (6) represents the excitation. When impulse excitation is used, E_s is set to 1 for $n = 0$, and 0 otherwise. Note here that the time stepping starts from $n = 0$ and E_s^0 appears two times both in steps n =0 and 1 in the calculation. Figure 3 shows calculated discrete Green functions when the terminating impedance R is set to 377 and 50Ω, respectively. A virtually semi-infinitely long mesh (1024Δx) was used in the calculation.

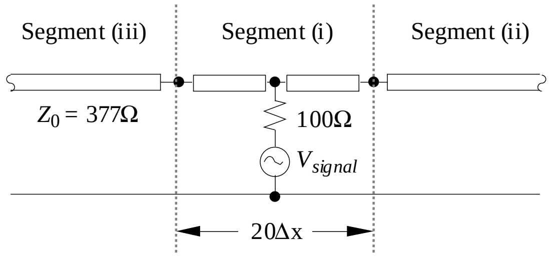

Fig. 1 One-dimensional example - a model of plane waves propagating in x-direction: infinite transmission line loaded with a lumped signal source.

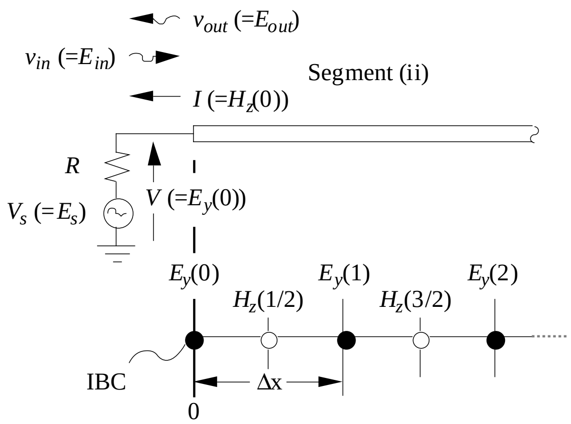

Fig. 2 Characterization of the segment (ii) by a discrete Green function.

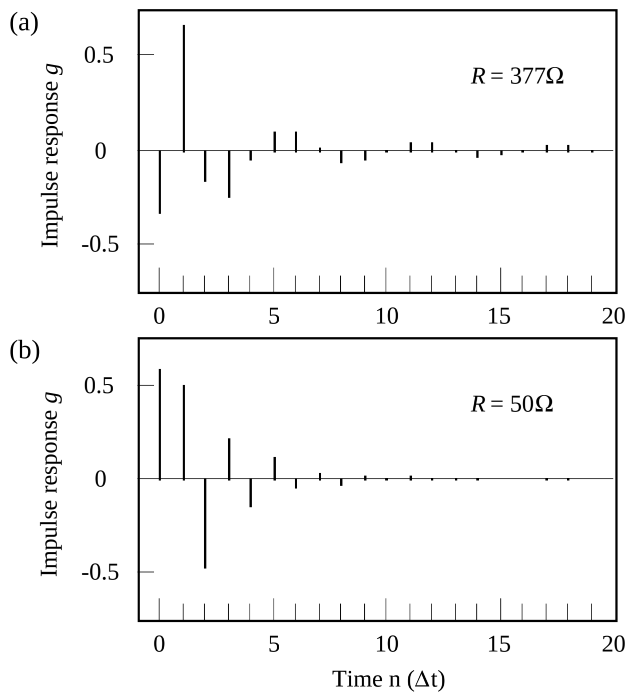

Fig. 3 Calculated discrete Green functions for the segment (ii): Δx/Δt = 2c; (a) R = 377Ω; (b) R = 50Ω.

II-II. Convolution

Once the Green function is obtained, the response v_{out} (=E_{out}) of segment (ii) for arbitrary input v_{in} (=E_{in}) can be calculated by the convolution

$$E^n_{out} = \sum_{i=0}^{n} g^{n-i} E^i_{in} \quad . \tag{7}$$

The response of segment (iii) can be written by (7) as well since the Green function of (iii) is identical to that of (ii). Next, we incorporate Eq. (7) into the calculation of segment (i) as the boundary condition at the removed nodes. Figure 4 depicts the interface between segments (i) and (ii). The removed node of segment (i) is terminated by a voltage source E_s with the standard impedance R, too. Then the influence of segment (ii) can be fed back via voltage source E_s. Taking into account that $E_{2,in} = E_{1,out}$ and $E_{2,out} = E_{1,in}$,

$$E^{n-1}_{1,in} = \sum_{i=0}^{n-1} g^{n-1-i} E^i_{1,out} \quad , \tag{8}$$

$$E^n_{1,in} = g^0 E^n_{1,out} + \sum_{i=0}^{n-1} g^{n-i} E^i_{1,out} \quad . \tag{9}$$

Recalling the relations given by Eqs. (4) and (5), we notice that Eq. (8) represents $E_s^{n-1/2}$, and that Eq. (9) can be modified to

$$E^n_s = \frac{2g^0}{1+g^0} E^n_y(0) + \frac{2}{1+g^0} \sum_{i=0}^{n-1} g^{n-i} E^i_{1,out} \quad . \tag{10}$$

Substituting these results into the excitation term of Eq. (6), we eventually get

$$\begin{aligned} E^n_y(0) = {} & \frac{\varepsilon_0 \varepsilon_{ry} - \Delta t / R\Delta x}{\varepsilon_0 \varepsilon_{ry} + \Delta t\{1-2g^0/(1+g^0)\}/R\Delta x} E^{n-1}_y(0) \\ & + \frac{2\Delta t}{[\varepsilon_0 \varepsilon_{ry} + \Delta t\{1-2g^0/(1+g^0)\}/R\Delta x]\Delta x} H^{n-1/2}_z(-1/2) \\ & - \frac{2\Delta t}{[\varepsilon_0 \varepsilon_{ry} + \Delta t\{1-2g^0/(1+g^0)\}/R\Delta x]R\Delta x} \\ & \sim \left(\frac{1}{1+g^0} \sum_{i=0}^{n-1} g^{n-i} E^i_{1,out} + \sum_{i=0}^{n-1} g^{n-1-i} E^i_{1,out} \right) \end{aligned} \tag{11}$$

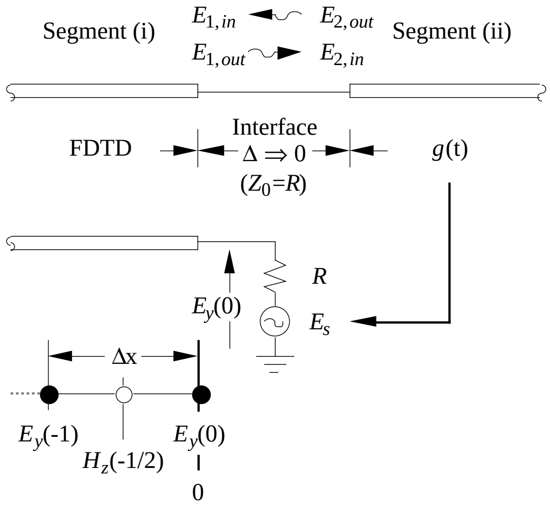

Fig. 4 Replacement of the segment (ii) by the Green function $g(t)$.

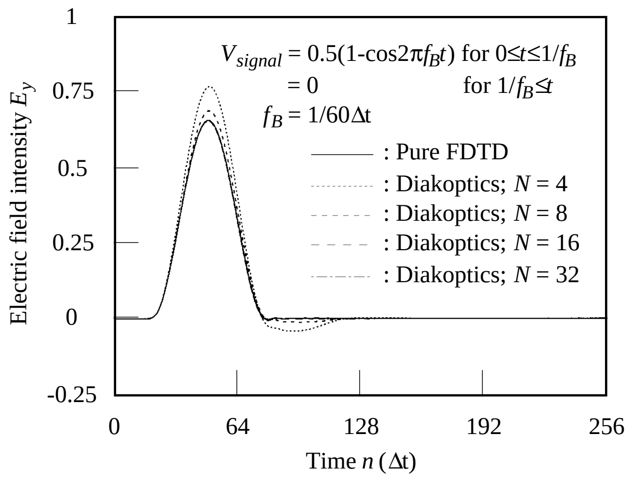

Fig. 5 Simulation result of example in Fig. 1: waveforms at the interface between the segments (i) and (ii). Pure FDTD uses a virtually infinitely long mesh ((i)+(ii)+(iii) = 1024Δx). In the diakoptics, R is set to 50Ω. N is number of terms in the convolution.

for updating value $E_y(0)$ of the removed node of segment (i). Directional wave $E_{1,out}$ has to be evaluated at every time step since it is required in the next updating of $E_y(0)$ as seen in Eq. (11). This can be done according to

$$\begin{aligned} E^n_{1,out} &= E^n_y(0) + \frac{1}{2} E^n_s \\ &= \frac{1}{1+g^0} \left(E^n_y(0) + \sum_{i=0}^{n-1} g^{n-i} E^i_{1,out} \right) \end{aligned} \quad . \tag{12}$$

Consequently, the update of all necessary variables can be obtained successfully. Figure 5 presents a result for the circuit shown in Fig. 1. Waveforms calculated by the diakoptics are compared with the result of a pure FDTD analysis. In the diakoptics, the number of terms in the convolution [Eqs. (8) and (9)] are varied as parameter N to see if we can truncate the Green function by an appropriate length. Good agreement with the pure FDTD was obtained by taking the first 32 terms of the discrete Green function into account in this particular example.

III. EXTENSION TO 2- AND 3-DIMENSIONAL PROBLEMS

Extension to multi-port circuits and/or 2- and 3-dimensional problems is straightforward. In these cases, we handle a lot of removed nodes for each segment which represent tangential electric field components on the interface planes. Therefore, a set of Green functions that includes all combination of impulse responses between the removed nodes is required. However, since g^0 is always zero when the observation point is different from the excitation point, formulas (11) and (12) do not formally change so much. Only the terms including the summation by Σ need to be modified.

Figure 6 presents a simulation result for a 2-dimensional case. The rectangular waveguide with an inductive iris shown in (a) was analyzed using the pure FDTD and the diakoptics. The structure was soft-excited at the interface plane between segments (i) and (iii) by a band-limited raised cosine pulse with the profile of the TE_{10} mode. Then the waveform was compared at the observation point over the iris. The waveforms compare precisely within 5 digits of accuracy.

IV. CONCLUSIONS

We have proposed a new interface for the FDTD diakoptics. The formulation was given with 1- and 2-dimensional numerical examples, which demonstrate the validity of the interface. This interface emulates the TLM type interface without using an absorbing boundary condition, and thus is capable of analyses that are as efficient as those of the TLM method.

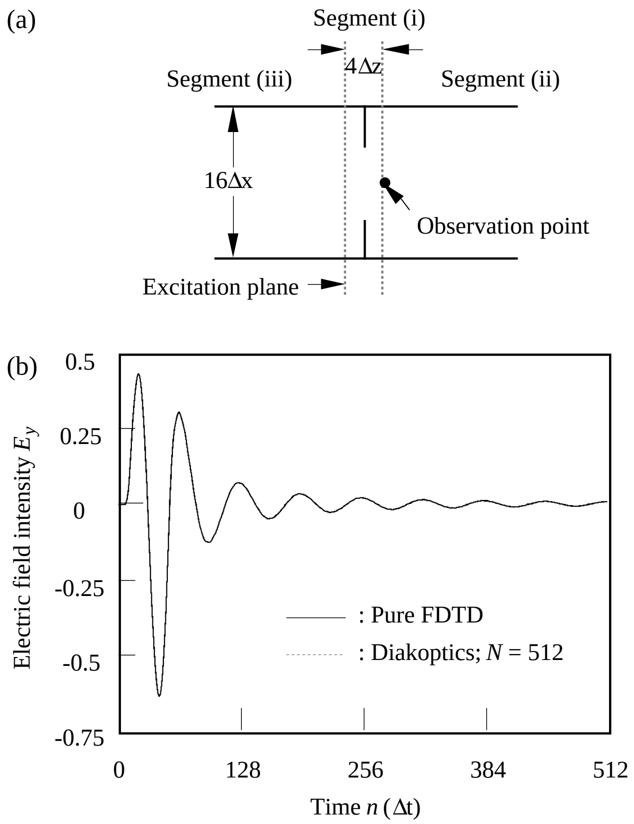

Fig. 6 Comparison of the results for two-dimensional problem between the pure FDTD and the diakoptics.

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